

Rectangular Hollow Structural Section Properties

Given the basic dimensions of a rectangular hollow structural section (HSS) shown below, this four part procedure:

Part 1: Computes its cross-sectional properties. Although standard HSS cross-section properties are available from other sources, engineers who optimize dynamically loaded structures will find this procedure of interest, as it allows input of actual, or worst case¹, dimensions.

Part 2: Recomputes the cross-sectional properties using the same equations, only this time the integral terms have been symbolically evaluated into an algebraic form for ease of calculation and/or program implementation.

Part 3: Generates a *.csv file containing midplane corner radii coordinates that define the given rectangular HSS cross-section with reasonable accuracy. This data is written in a format specifically for importing into the "General Section Type" input menu of Algor's Mechanical Event Simulation software. The points are also plotted to allow for visual verification.

Part 4: Solves beam equations with the calculated section properties. These results may be used to verify that the input parameters of a comparable finite element model have been correctly specified. Once the analyst has thereby gained confidence, he may proceed adding complexity to his model.

While the procedure presented herein utilizes basic equations for computing beam cross-sectional properties, and is believed to compute exact results, the analyst is advised to:

- 1) Carefully check that all input information and assumptions apply to the given application, as well as;
- 2) Compare results with property information published by the American Institute of Steel Construction, Inc.

The results from this procedure should not be used or relied upon for any specific application without verification of its accuracy, suitability and applicability, by a licensed professional engineer, designer or architect, as the author shall not be liable for incidental or consequential damages in connection with, or arising out of its use.

Basic Input Parameters:

Height, Section.....:	$h := 8$	$h = 8.000$	in.
Width, Section.....:	$b := 4$	$b = 4.000$	in.
Wall Thickness, Nominal.....:	$t_n := 0.5$	$t_n = 0.500$	in.
Corner Radii, Outside ²:	$r_o := 2 \cdot t_n$	$r_o = 1.000$	in.
Wall Thickness, Design ³:	$t := 0.93 \cdot t_n$	$t = 0.4650$	in.

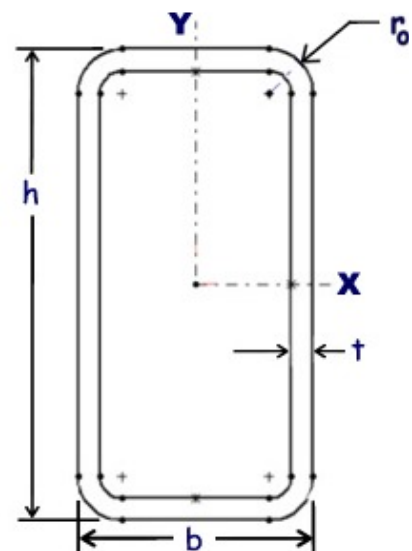


Figure 1: Basic Dimensions

Corner Radii Coordinates:

Horizontal Flat Dimension⁴.....: $x := b - 2 \cdot r_o$ $x = 2.0000$ in.

Vertical Flat Dimension.....: $y := h - 2 \cdot r_o$ $y = 6.0000$ in.

Midplane Radius.....: $r := r_o - \frac{t}{2}$ $r = 0.7675$ in.

Point 1, Horizontal Coordinate: $x1 := \frac{x}{2} + r$ $x1 = 1.7675$

Point 1, Vertical.....Coordinate: $y1 := \frac{y}{2}$ $y1 = 3.0000$ in.

Point 2, Horizontal Coordinate: $x2 := \frac{x}{2} + \frac{r}{\sqrt{2}}$ $x2 = 1.5427$ in.

Point 2, Vertical.....Coordinate: $y2 := \frac{y}{2} + \frac{r}{\sqrt{2}}$ $y2 = 3.5427$ in.

Point 3, Horizontal Coordinate: $x3 := \frac{x}{2}$ $x3 = 1.0000$ in.

Point 3, Vertical.....Coordinate: $y3 := \frac{y}{2} + r$ $y3 = 3.7675$ in.

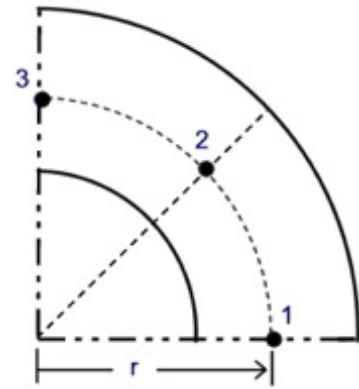


Figure 2: Midplane Corner Points

PART 1 - Section Properties:

For ease of recognizing how each term in the following equations corresponds with segments of a rectangular HSS cross-section, the integrals have been left in their initial form. However, for ease of calculation and/or programming, these terms are symbolically evaluated into an algebraic form in Part 2 of this procedure.

Cross Sectional Area:

Inside Radius.....: $r_i := r_o - t$ $r_i = 0.5350$ in.

$$A := 2 \cdot t \cdot (x + y) + 2 \cdot \int_0^\pi \int_{r_i}^{r_o} r \, dr \, d\theta$$

A = 9.6824 in²

Second Moment of Area About X Axis:

Lower Limit of Integration.....: $\eta_1 := y3 - \frac{t}{2}$ $\eta_1 = 3.5350$

Upper Limit of Integration.....: $\eta_2 := y3 + \frac{t}{2}$ $\eta_2 = 4.0000$

Sharp Corner Second Moment of Area.: $I_{xx} := \frac{b \cdot h^3}{12} - \frac{(b - 2 \cdot t) \cdot (h - 2 \cdot t)^3}{12}$ $I_{xx} = 80.2569$ in⁴

Accurate Second Moment of Area.:

$$I_x := 2 \cdot x \cdot \int_{\eta_1}^{\eta_2} y^2 \, dy + 4 \cdot t \cdot \int_0^{y1} y^2 \, dy + 2 \cdot \int_0^\pi \int_{r_i}^{r_o} r \cdot \left(\frac{y}{2} + r \cdot \sin(\theta) \right)^2 \, dr \, d\theta$$

I_x = 70.8520 in⁴

Section Modulus About X Axis:

Section Modulus About.....: $S_x := \frac{2 \cdot I_x}{h}$ $S_x = 17.7130$ in³

Second Moment of Area About Y Axis:

Lower Limit of Integration.....: $\xi_1 := x_1 - \frac{t}{2}$ $\xi_1 = 1.5350$

Upper Limit of Integration.....: $\xi_2 := x_1 + \frac{t}{2}$ $\xi_2 = 2.0000$

Sharp Corner Second Moment of Area.: $I_{yy} := \frac{b^3 \cdot h}{12} - \frac{(b - 2 \cdot t)^3 \cdot (h - 2 \cdot t)}{12}$ $I_{yy} = 25.619$ in⁴

Accurate Second Moment of Area.:

$$I_y := 4 \cdot t \cdot \int_0^{x_3} x^2 dx + 2 \cdot y \cdot \int_{\xi_1}^{\xi_2} x^2 dx + 2 \cdot \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{r_i}^{r_o} r \cdot \left(\frac{x}{2} + r \cdot \cos(\theta) \right)^2 dr d\theta$$
 $I_y = 23.3745$ in⁴

Section Modulus About Y Axis:

Section Modulus About Y-Y Axis.....: $S_y := \frac{2 \cdot I_y}{b}$ $S_y = 11.6873$ in³

Torsion Constant & Second Polar Moment of Area:

The Torsion Constant, J, for a thin wall HSS member in pure torsion is calculated. Please refer to Spyrakos 1997, page 113 for further comments on the restrictions associated with the use this equation.

In that the Torsion Constant may be confused with the Second Polar Moment of Area, it too is calculated for comparison. They are only equal for circular sections.

Mid Wall Depth.....: $h_m := h - t$ $h_m = 7.5350$ in

Mid Wall Breadth.....: $b_m := b - t$ $b_m = 3.5350$ in

Torsion Constant, assuming sharp corners: (Syrakos 1997, pg 113).....: $J := \frac{2 \cdot b_m^2 \cdot h_m^2 \cdot t}{b_m + h_m}$ $J = 59.605$ in⁴

Torsion Constant (Salmon 1996, pg 461, Eq. 8.10.12).....: $J := \frac{4 \cdot t \cdot [y \cdot b_m + (2 \cdot r \cdot x) + \pi \cdot r^2]^2}{2 \cdot (x + y + \pi \cdot r)}$ $J = 60.993$ in⁴

Second Polar Moment of Area.....: $J_o := I_x + I_y$ $J_o = 94.227$ in⁴

Shear Area:

The areas that resist shear forces in the **X** and **Y** directions are assumed to be as shown in figures 3a and 3b, respectively. The appropriateness of this assumption for a given application is the user's responsibility. Shear areas obtained from the equations in Spyrakos, 1996 are also calculated. However, for them to compare with results obtained in Algor's Cross-Section Library, the 0.93 wall thickness reduction factor in the input should be changed to 1.00.

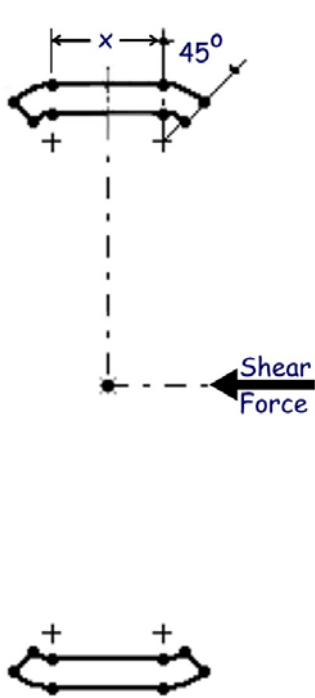


Figure 3a: Force in **X** direction.

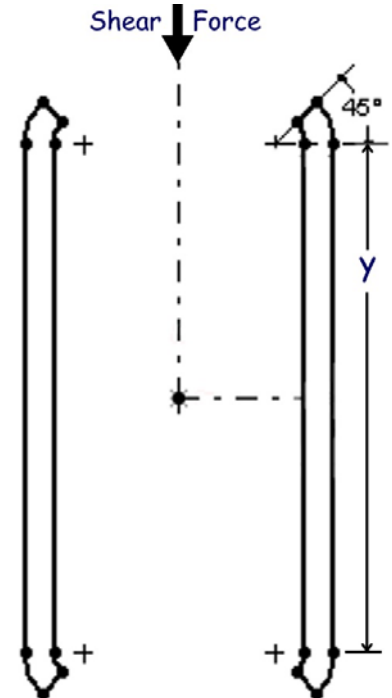


Figure 3b: Force in **Y** direction.

Shear Area, Y direction.....: (Syrakos 1996, pg 37)	$S_{ay} := 2 \cdot t \cdot h$	$S_{ay} = 7.440$	in^2
Shear Area, X direction.....: (Syrakos 1996, pg 37)	$S_{ax} := 2 \cdot t \cdot b$	$S_{ax} = 3.720$	in^2
Shear Area, Y direction.....:	$S_{ay} := t \cdot (2 \cdot y + \pi \cdot r)$	$S_{ay} = 6.701$	in^2
Shear Area, X direction.....:	$S_{ax} := t \cdot (2 \cdot x + \pi \cdot r)$	$S_{ax} = 2.981$	in^2
Check - Should Equal Zero.....:	$S_{ax} + S_{ay} - A = 0.0000$		

PART 2 - Section Properties - (closed form equations)

For ease of obtaining numerical results and for programing, the integrals terms in the above equations have been evaluated. Also, the **X, Y** coordinates have been replaced by x3, x2 local coordinates to allow straight forward interpretation of results by Algor users.

$$A := 2 \cdot t \cdot (x + y) + \pi \cdot r_o^2 - \pi \cdot r_i^2 \quad \boxed{A = 9.68239} \quad \text{in}^2$$

$$J1 := \frac{4 \cdot t \cdot [y \cdot b_m + (2 \cdot r \cdot x) + \pi \cdot r^2]^2}{2 \cdot (x + y + \pi \cdot r)} \quad \boxed{J1 = 60.99317} \quad \text{in}^4$$

$$I2_Term1 := \frac{4}{3} \cdot t \cdot x^3 + 2 \cdot y \cdot \left(\frac{1}{3} \cdot \xi_2^3 - \frac{1}{3} \cdot \xi_1^3 \right)$$

$$I2_Term2 := \frac{1}{4} \cdot \pi \cdot r_o^4 + \frac{4}{3} \cdot x \cdot r_o^3 + \frac{1}{4} \cdot x^2 \cdot r_o^2 \cdot \pi - \frac{1}{4} \cdot \pi \cdot r_i^4 - \frac{4}{3} \cdot x \cdot r_i^3 - \frac{1}{4} \cdot x^2 \cdot r_i^2 \cdot \pi$$

$$I2 := I2_Term1 + I2_Term2 \quad \boxed{I2 = 23.37454} \quad \text{in}^4$$

$$I3_Term1 := 2 \cdot x \cdot \left(\frac{1}{3} \cdot \eta_2^3 - \frac{1}{3} \cdot \eta_1^3 \right) + \frac{4}{3} \cdot t \cdot y^3$$

$$I3_Term2 := \frac{1}{4} \cdot \pi \cdot r_o^4 + \frac{4}{3} \cdot y \cdot r_o^3 + \frac{1}{4} \cdot y^2 \cdot r_o^2 \cdot \pi - \frac{1}{4} \cdot \pi \cdot r_i^4 - \frac{4}{3} \cdot y \cdot r_i^3 - \frac{1}{4} \cdot y^2 \cdot r_i^2 \cdot \pi$$

$$I3 := I3_Term1 + I3_Term2 \quad \boxed{I3 = 70.85198} \quad \text{in}^4$$

$$S2 := \frac{2 \cdot I2}{b} \quad \boxed{S2 = 11.68727} \quad \text{in}^3$$

$$S3 := \frac{2 \cdot I3}{h} \quad \boxed{S3 = 17.71300} \quad \text{in}^3$$

$$SA2 := t \cdot (2 \cdot y + \pi \cdot r) \quad \boxed{SA2 = 6.70120} \quad \text{in}^2$$

$$SA3 := t \cdot (2 \cdot x + \pi \cdot r) \quad \boxed{SA3 = 2.98120} \quad \text{in}^2$$

PART 3 - Generate CSV Data File:

The mid-plane corner radii coordinates, that reasonably define the HSS cross-section, are now assigned to a matrix formatted for importing into the "General Section Type" input menu of Algor's Mechanical Event Simulation software. The data is then written to a *.csv file.

This procedure has been successfully used with previous versions of the Algor software. However, as this article is being released, an unresolved error message is generated when checking the FEA model within FEMPRO, release 16.0. When this issue is resolved, the data arrangement in either of the matrices below may need to be modified to accommodate any changes made to the software. Or, the analyst may opt to edit the data within a Microsoft Excel spreadsheet.

$$\begin{array}{l}
 \text{Section Number.....:} \quad s := 1 \\
 xy := \begin{pmatrix} t & -x3 & y3 & t & x3 & y3 & 0 & 0 & 0 \\ t & x3 & y3 & t & x2 & y2 & t & x1 & y1 \\ t & x1 & y1 & t & x1 & -y1 & 0 & 0 & 0 \\ t & x1 & -y1 & t & x2 & -y2 & t & x3 & -y3 \\ t & x3 & -y3 & t & -x3 & -y3 & 0 & 0 & 0 \\ t & -x3 & -y3 & t & -x2 & -y2 & t & -x1 & -y1 \\ t & -x1 & -y1 & t & -x1 & y1 & 0 & 0 & 0 \\ t & -x1 & y1 & t & -x2 & y2 & t & -x3 & y3 \end{pmatrix} \\
 xy1 := \begin{pmatrix} s & t & -x3 & y3 & t & x3 & y3 & 0 & 0 & 0 \\ s & t & x3 & y3 & t & x2 & y2 & t & x1 & y1 \\ s & t & x1 & y1 & t & x1 & -y1 & 0 & 0 & 0 \\ s & t & x1 & -y1 & t & x2 & -y2 & t & x3 & -y3 \\ s & t & x3 & -y3 & t & -x3 & -y3 & 0 & 0 & 0 \\ s & t & -x3 & -y3 & t & -x2 & -y2 & t & -x1 & -y1 \\ s & t & -x1 & -y1 & t & -x1 & y1 & 0 & 0 & 0 \\ s & t & -x1 & y1 & t & -x2 & y2 & t & -x3 & y3 \end{pmatrix}
 \end{array}$$

The above matrix format worked with earlier MES versions.

Write data to a *.csv file.....:
 (The name and location of this file may be changed within Mathcad by right clicking the floppy disk icon, then selecting "Choose File").

 C:\8x4x0-500.csv

xy1

xy1 =

	0	1	2	3	4	5	6	7	8	9
0	1.000	0.465	-1.000	3.768	0.465	1.000	3.768	0.000	0.000	0.000
1	1.000	0.465	1.000	3.768	0.465	1.543	3.543	0.465	1.768	3.000
2	1.000	0.465	1.768	3.000	0.465	1.768	-3.000	0.000	0.000	0.000
3	1.000	0.465	1.768	-3.000	0.465	1.543	-3.543	0.465	1.000	-3.768
4	1.000	0.465	1.000	-3.768	0.465	-1.000	-3.768	0.000	0.000	0.000
5	1.000	0.465	-1.000	-3.768	0.465	-1.543	-3.543	0.465	-1.768	-3.000
6	1.000	0.465	-1.768	-3.000	0.465	-1.768	3.000	0.000	0.000	0.000
7	1.000	0.465	-1.768	3.000	0.465	-1.543	3.543	0.465	-1.000	3.768

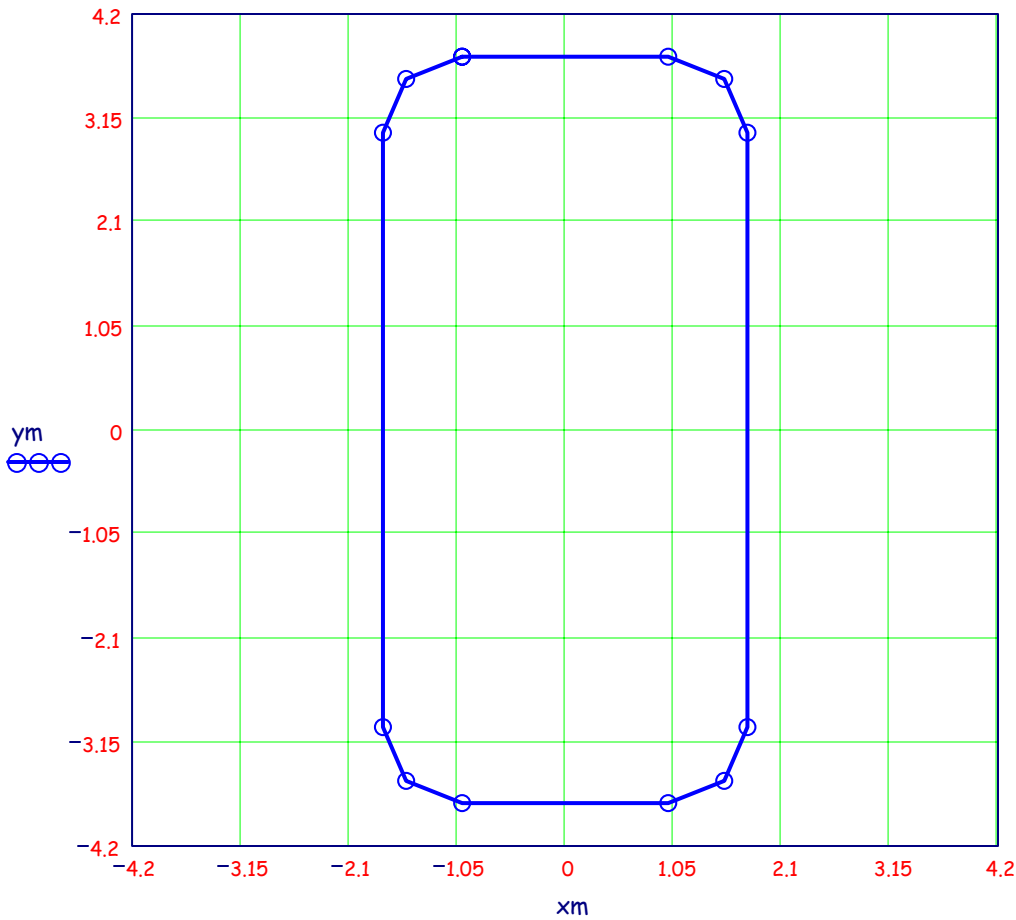
The coordinates are also assigned to two arrays and subsequently plotted to allow for ready visual verification. It will be necessary to manually adjust the minimum and maximum abscissa and ordinate values on the plot to accommodate larger sections.

The analyst may also wish to manually introduce small variations into the array values (or possibly use additional points at the mid-sides) to investigate other worst case tolerance effects such as the squareness or straightness of the sides.

```

xm := (xy0,1)
      (xy0,4)
      (xy1,4)
      (xy2,1)
      (xy2,4)
      (xy3,4)
      (xy4,1)
      (xy4,4)
      (xy5,4)
      (xy6,1)
      (xy6,4)
      (xy7,4)
      (xy7,7)

ym := (xy0,2)
      (xy0,5)
      (xy1,5)
      (xy2,2)
      (xy2,5)
      (xy3,5)
      (xy4,2)
      (xy4,5)
      (xy5,5)
      (xy6,2)
      (xy6,5)
      (xy7,5)
      (xy7,8)
  
```



PART 4 - Example Beam Deflection & Stress Results

Given: Length, Applied Load, Elastic Modulus, and Section Properties (calculated above).

Length, Beam.....: $L := 144$ $L = 144.0$ in.

Load, Total Applied: $F := 250$ $F = 250$ lbf

Elastic Modulus.....: $E := 29 \cdot 10^6$ $E = 29.0 \times 10^6$ psi

Section-Properties:

$I_x = 70.852$ in⁴ $I_y = 23.375$ in⁴

$S_x = 17.713$ in³ $S_y = 11.687$ in³

Determine: For Pin-Pin HSS Beam with Central Point Load.

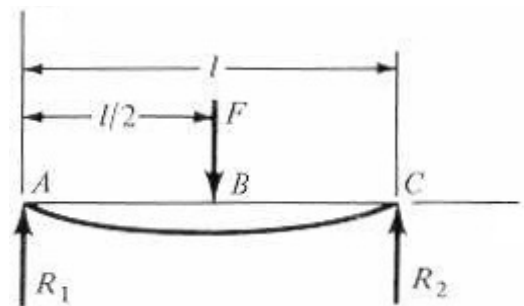
Stress & Deflection about the x axis:

$$x_{\max} := \frac{-F \cdot L^3}{48 \cdot E \cdot I_x} \qquad x_{\max} = -0.00757 \text{ in}$$

For $x \geq L/2$:

$$M(x) := \frac{F}{2} \cdot (L - x) \qquad M\left(\frac{L}{2}\right) = 9000.0 \text{ in} \cdot \text{lbf}$$

$$\sigma(x) := \frac{M(x)}{S_x} \qquad \sigma\left(\frac{L}{2}\right) = 508 \text{ psi}$$



Stress & Deflection about the y axis:

$$y_{\max} := \frac{-F \cdot L^3}{48 \cdot E \cdot I_y} \qquad y_{\max} = -0.02294 \text{ in}$$

$$M(x) := \frac{F}{2} \cdot (L - x) \qquad M\left(\frac{L}{2}\right) = 9000 \text{ in} \cdot \text{lbf}$$

$$\sigma(x) := \frac{M(x)}{S_y} \qquad \sigma\left(\frac{L}{2}\right) = 770 \text{ psi}$$

Given: Pin-Pin HSS Beam with Uniformly Distributed Load

Uniform Load.....: $w := \frac{F}{L}$ $w = 1.74$ lbf / in

Stress & Deflection about the x axis:

$$x_{\max} := \frac{-5 \cdot w \cdot L^4}{384 \cdot E \cdot I_x} \quad x_{\max} = -0.00473 \quad \text{in}$$

For $x \geq L/2$:

$$M(x) := \frac{w \cdot x}{2} \cdot (L - x) \quad M\left(\frac{L}{2}\right) = 4500.0 \quad \text{in} \cdot \text{lbf}$$

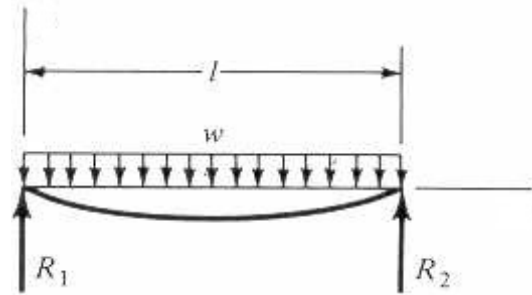
$$\sigma(x) := \frac{M(x)}{S_x} \quad \sigma\left(\frac{L}{2}\right) = 254 \quad \text{psi}$$

Stress & Deflection about the y axis:

$$y_{\max} := \frac{-5w \cdot L^4}{384 \cdot E \cdot I_y} \quad y_{\max} = -0.01434 \quad \text{in}$$

$$M(x) := \frac{w \cdot x}{2} \cdot (L - x) \quad M\left(\frac{L}{2}\right) = 4500.0 \quad \text{in} \cdot \text{lbf}$$

$$\sigma(x) := \frac{M(x)}{S_y} \quad \sigma\left(\frac{L}{2}\right) = 385 \quad \text{psi}$$



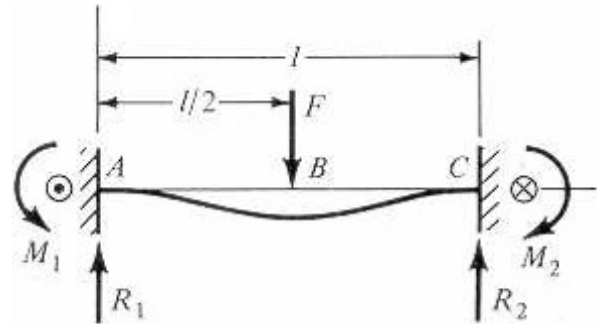
Determine: For Fixed-Fixed HSS Beam with Central Point Load.

Stress & Deflection about the x axis:

$$x_{mx} := \frac{-F \cdot L^3}{192 \cdot E \cdot I_x} \quad x_{mx} = -0.00189 \quad \text{in}$$

$$M(x) := \frac{F}{8} \cdot (4 \cdot x - L) \quad M\left(\frac{L}{2}\right) = 4500 \quad \text{in - lbf}$$

$$\sigma(x) := \frac{M(x)}{S_x} \quad \sigma\left(\frac{L}{2}\right) = 254 \quad \text{psi}$$



Stress & Deflection about the y axis:

$$y_{mx} := \frac{-F \cdot L^3}{192 \cdot E \cdot I_y} \quad y_{mx} = -0.00574 \quad \text{in}$$

$$M(x) := \frac{F}{8} \cdot (4 \cdot x - L) \quad M\left(\frac{L}{2}\right) = 4500.0 \quad \text{in - lbf}$$

$$\sigma(x) := \frac{M(x)}{S_y} \quad \sigma\left(\frac{L}{2}\right) = 385 \quad \text{psi}$$

¹Depending on the degree of optimization desired, the analyst may wish to examine what effect the variation of some dimensions (e.g. h, b, t, and r_o) within their tolerance bounds will have on the cross-sectional properties. The permissible dimensional variations may be found in the appropriate standards, eg. ASTM A500 and A501,...

²The outside corner radius is set as 2t for ERW HSS. However, it will vary based on the material, mill equipment and procedures. For example, extruded aluminum rectangular HSS frequently has small corner radii. For original designs, requiring only small material orders, obtaining specific corner radii data prior to actually receiving the order is often not practical.

Ref. "Manual of Steel Construction - Load and Resistance Factor Design", AISC, 3rd. ed., 2001, pg. 1-6.

³When specifying electric resistance-welded (ERW) tubing, the American Institute of Steel Construction (AISC) and the Steel Tube Institute of North America (STI) recommend that a wall thickness reduction factor of 0.93 be used for making design calculations. The applicable ASTM A500 standard for this tubing allows a plus/minus 10% variation from the nominal wall thickness. And, since this tubing is made from plate or strip, which can be held to a tighter thickness tolerance, United States mills consistently produce ERW HSS with a wall thickness near the lower bound of the tolerance.

Ref. "Manual of Steel Construction - Load and Resistance Factor Design", AISC, 3rd. ed., 2001, pg. 16.2-25.

⁴The **X** & **Y** are the local coordinates used in the AISI Manual of Steel Construction. Algor users may readily associate these with Algor's respective x3, x2 local coordinate designation. On the other hand, the x & y variables represent the respective HSS horizontal and vertical flat dimensions. To simplify the appearance of the equations within this procedure, these variables were not subscripted, which would have otherwise more clearly distinguished them from the local coordinates.